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# A NEW EMPIRICAL CORRELATION FOR FROST DENSITY PREDICTION OVER FLAT PLATE

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***Abstract.*** *A new correlation for frost density prediction is developed based on the experimental database found in the literature. A careful recognition of the main dimensionless groups was performed. The main dimensionless groups were considered in the development of the new empirical model. The experimental database cover flat surfaces with surface temperature range from -20.7°C to -5°C and air temperature range from 13.4°C to 23.4°C. Frost density literature models were also compared with the experimental database. The new correlation best predicted the database with a mean absolute error of 18.9 %.*
***Keywords:*** *Frost density, empirical model, cold surfaces.*

## 1. INTRODUCTION

The frost layer development is an undesirable process that affects a wide range of industrial fields like aviation, refrigeration, and HVAC (heating, ventilation, and air condition) systems (Song and Dang, 2018). Specific conditions of temperature and humidity are necessary to begin the frosting phenomenon. This development occurs when a moist air found a cold surface, with both dew-point and plate surface temperatures are below 0°C. Thus, the desublimation takes places, i.e., the water vapor change phase directly to the solid phase. Furthermore, the frost layer can also be developed by the freezing of condensed water over a cold surface (Piucco et al. 2018). In summary, the frost layer is dynamic heat- and mass- transfer, transient, moving-boundary and non-linear and variable-density physical phenomenon (Song and Dang, 2018) that has been intensely investigated in last few decades.

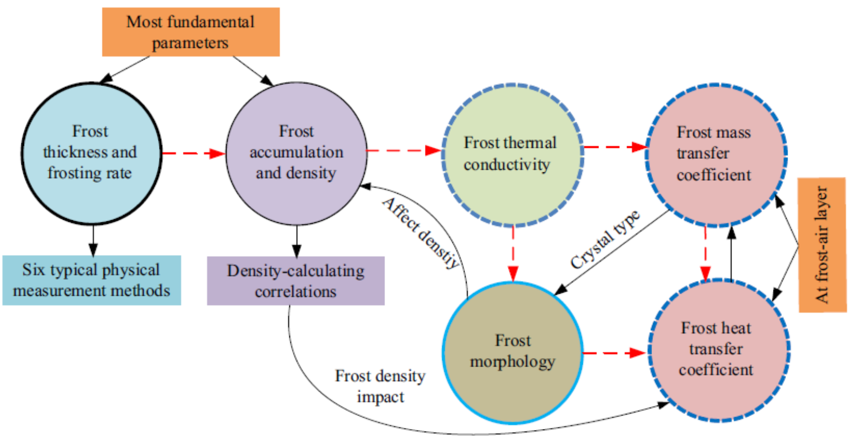



Figure 1. Roadmap presented in Song and Dang (2018).

The frost growth over evaporator affected the operation of the refrigeration system. Hence, the frost layer decreases the heat transfer and blocks the airflow. The improvement of the efficiency of the refrigeration system is enhanced through defrosting techniques. So, the frosting/defrosting process is needed to be investigated in the last decade. The most fundamental parameter in the frosting process is the (i) frost thickness (and frosting rate) and (ii) frost density (frost accumulation). Indeed, several parameters such as frost thermal conductivity, frost mass transfer coefficient, and frost heat transfer coefficient are affected by frost thickness and frost density. In Fig 1, the roadmap of the main parameters that drive the frosting process is presented (Song and Dang, 2018). The present work aims to investigate the performance of frost density models found in the literature.

Several models for frost density ($\rho_f$) prediction have been developed along the last decades. An early correlation was proposed by Hosoda and Uzuhashi (1967), which depended on wall temperature ($T_w$) and air velocity ($v_a$), as described by Eq.(1). Hayashi et al. (1977) also introduced the classical model given by Eq.(2) to predicted frost density. However, the model developed by Hayashi et al. (1977) was a function of the frost surface temperature ($T_s$), which is parameter not easily determined. So, the frosted surface temperature dependence is a drawback of this correlation.

$$\rho_f = 340 \left| T_W \right|^{-0.445} + 85 v_a \quad (1)$$

$$\rho_f = 650 \exp\left(0.277 \mathrm{T}_s\right) \quad (2)$$

Mao (1991) investigated the frost formation over a flat surface. This work investigated the frost thickness, frost mass concentration, frost density, frost thermal conductivity and heat and mass transfer along the flat plate. The measurement of frost density was performed using flush-mounted removable disks which were done at specific distances from leading edge. Mao et al. (1992) developed a dimensionless frost density ( $\rho_f^*$ ) correlation given by Eq. (3). The local frost density ($\rho_f$) is obtained from Eq. (3.1), where ($\rho_i$) is the ice density ($\rho_i$= 920 kg/m³). The dimensionless position ($X^*$) is defined by Eq. (3.2), where $D_h$ is the hydraulic diameter at the inlet of the test section. The Reynolds number ($Re_D$) and Fourier number are determined from Eq. (3.3) and Eq. (3.4), respectively. Both dimensionless groups used the hydraulic diameter ($D_h$) as characteristic length. The wall temperature ($T_w$), the air temperature ($T_a$), the triple point of water ($T_{tp}$) and humidity ratio ($\omega_a$) are taken in to account in the correlation presented by the authors.

$$\rho_f^* = 5.55910^{-6} \left(X^*\right)^{-0.137} \omega_a^{-0.413} \left(\frac{T_{tp} - T_w}{T_a - T_w}\right)^{0.997} \mathrm{Re}_D^{0.715} Fo^{0.252} \quad (3)$$

$$\rho_f^* = \frac{\rho_f}{\rho_i} \quad (3.1)$$

$$X^* = \frac{x}{D_h} \quad (3.2)$$

$$\mathrm{Re}_D = \frac{v_a D_h}{\nu_a} \quad (3.3)$$

$$Fo = \frac{\alpha_a t}{\left(D_h\right)^2} \quad (3.4)$$

Yang and Lee (2004) also developed a dimensionless correlation given by Eq. (4) to predict the frost density over the cold plate. However, the new correlation proposed neglect the variation of frost density with the length of the flat plate. The Reynolds number and Fourier number are also presented in Yang and Lee (2004), however, both dimensionless numbers adopted the plate length as characteristic length. A new version of Hayashi et al. (1977)'s correlation was presented by Hermes et al. (2009), which accounted the wall temperature ($T_w$) as observed in Eq. (5). The improvement suggested by Hermes et al. (2009) enhanced the model, providing more accurate predictions. However, the correlation still is based on frost surface temperature ($T_s$).

$$\rho_f^* = 1.5410^{-4} \mathrm{Re}^{0.351} Fo^{0.311} \omega_a^{-0.368} \left(\exp\left(\frac{T_a - T_{tp}}{T_a - T_w}\right)\right)^{2.4} \quad (4)$$

$$\rho_f = 270 \exp\left(0.266 \mathrm{T}_s - 0.0615 T_w\right) \quad (5)$$

Kandula (2012) also developed a dimensionless correlation for frost density prediction defined by Eq. (6). This dimensionless model was proposed for laminar forced flow over a flat surface and has a strong dependence on frost surface temperature ($T_s$). The correlation neglected the air parameters (air temperature, air humidity, and air velocity). Indeed, the author indirectly accounts for these parameters through the frost surface temperature ($T_s$), which is strongly affected by the air temperature ($T_a$) and air humidity. The critical Reynolds number for laminar-turbulent transition ( $Re_c \approx 10^5$ ) and the melting temperature ($T_m$) are also correlated with the dimensionless frost density presented by Kandula (2012).

$$\rho_f^* = 0.5\left(\frac{T_s - T_w}{T_m - T_w}\right)\exp\left(-\left(0.376 + 1.5\left(\frac{T_m - T_s}{T_m - T_w}\right)\right)\left(1 - \sqrt{\frac{Re}{Re_c}}\right)\right) \tag{6}$$

A semi-empirical correlation was developed by Hermes et al. (2014), which used the modified Jakob number (Hermes, 2012) defined by Eq. (7.1). The modified Jakob number (Ja) depends on the latent heat of desublimation of water ($i_{sv}$), humidity ratio difference ($\omega_a$ - $\omega_{sat,w}$) and the difference between the wall temperature ($T_w$) and air temperature saturation ($T_{sat,a}$). From the experimental database presented by Hermes et al. (2009), Hermes et al. (2014) developed the Eq. (7), which presented an accurate prediction according to the authors. Leoni et al. (2016) also developed a correlation based on the modified Jakob number (Ja) as observed by Eq. (8). Additionally, the model proposed by the author take in to account the Reynolds number (Re), time (t), and humidity ratio ($\omega_{sat,w}/\omega_a$). The newest correlation was adjusted with the experimental database found in the literature.

$$\rho_f = 2.2 Ja^{-3/2}\sqrt{t} \tag{7}$$

$$Ja = \frac{c_p\left(T_{sat,a} - T_w\right)}{i_{sv}\left(\omega_a - \omega_{sat,w}\right)} \tag{7.1}$$

$$\rho_f = 5.47\, Re^{0.16}\, Ja^{0.29} \frac{\omega_w}{\omega_a}^{0.61} t^{0.34} \tag{8}$$

Despite the recent developements of the empirical and semi-empirical correlation in the prediction of frost density, they are still not able to provide accurate results over a wide range of conditions. Leoni et al. (2017) acquired a new experimental database on a flat surface and evaluated the performance of several correlations found in the literature. In Fig. 2, the measured frost density is compared against prediction for correlation presented in the present work. No correlation was able to properly predict frost density database obtained for Leoni et al. (2017). So, the results suggest that more investigation is necessary to enhance the frost density prediction.

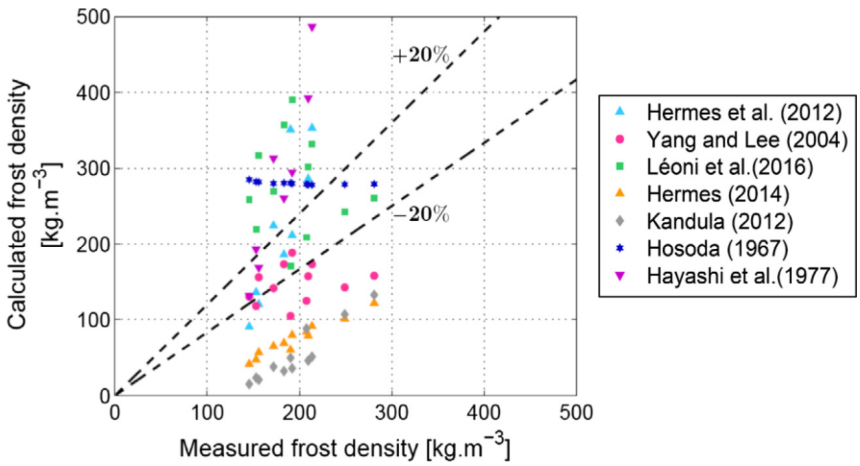



Figure 2. Frost density prediction presented by Leoni et al. (2017).

## 2. NEW CORRELATION FOR FROST DENSITY PREDICTION

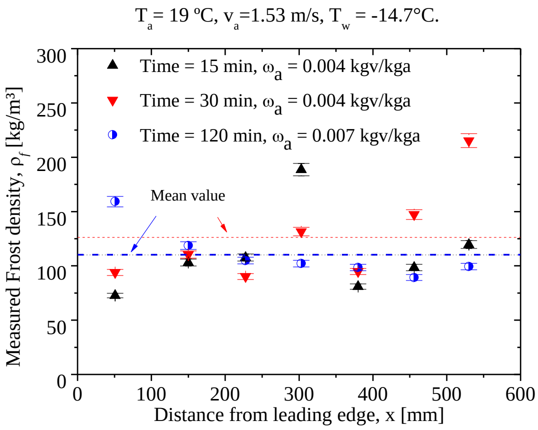
The performance of frost density correlations found in the literature suggested that a new correlation is needed to improve the predictions (Fig.2). Thus, the present work proposes to develop an empirical correlation based on the experimental database for flat plate found in the literature. The database is obtained from Mao (1991), Lee et al. (2003) and Hermes et al. (2009). The limitation of each database is presented in Table 1.

Table 1. Experimental database for frost density obtained from the literature.

| Experimental data source | | | Experimental conditions | | | |
|---|---|---|---|---|---|---|
| Reference | Symbol | Number of points | Operational limit [m/s] | $T_w$ [(1)], [°C] | $T_a$ [(2)], [°C] | L[(3)] [m] |
| Mao (1991) |  | 308 | $1.15 \le v_a \le 2.67$ | $-20.7 \le T_w \le -9.6$ | $13.4 \le T_a \le 23.4$ | 0.6 |
| Lee et al. (2003) |  | 21 | $1.75 \le v_a \le 2.5$ | $-20 \le T_w \le -10$ | $5 \le T_a \le 20$ | 0.3 |
| Hermes et al. (2009) |  | 24 | $v_a = 0.7$ m/s | $-15 \le T_w \le -5$ | $16 \le T_a \le 22$ | 0.1 |

wall temperature, [2]air temperature, [3]plate lengths

a)




b)

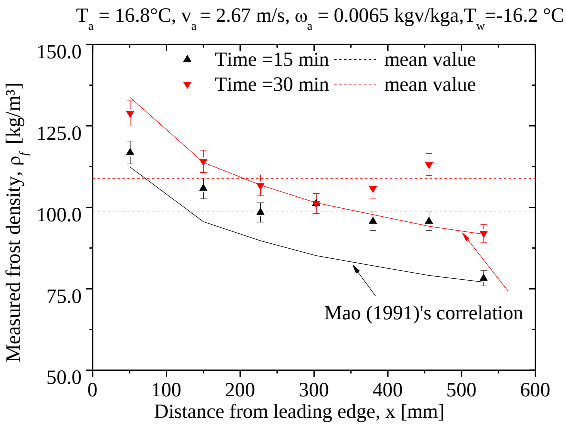
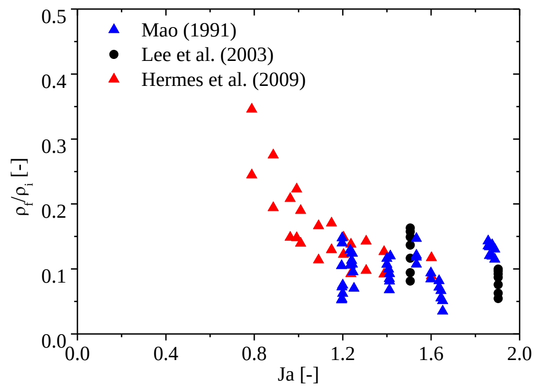



Figure 3. Measured frost density as a function of the distance from leading edge presented by Mao (1991).

a)

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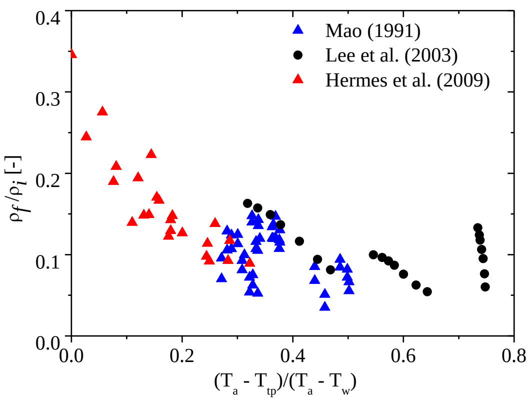



Figure 4. Dimensionlees frost density ( $\rho_f^*$ ) as a function of a) modified Jakob number (*Ja*) and b) dimensionles temperature (T*).

A careful evaluation of the frost density experimental database presented by Mao (1991) showed no clear trend with the distance from the leading edge (Fig. 3). In Fig.3a, it presented the frost density as a function of the distance of the leading edge for the condition: $T_a$= 19°C (air temperature), $T_w$ = -14.7 °C (plate surface temperature or wall temperature), and $v_a$ =1.53 m/s (air velocity). No trend with distance from the leading edge is found in Fig.3a. The same performance is obtained in Fig. 3b. ($T_a$= 16.8°C, $T_w$ = -16.2°C, $v_a$ = 2.67 m/s). Regarding the lack of a clear correlation of frost density with the distance from the leading edge (Fig. 3 and Fig. 3b), the proposed correlation dropped this parameter. Therefore, the average frost density for a given time is the average of seven measurements performed at along several positions on a flat plate (51 mm, 150 mm, 227 mm, 303 mm, 380 mm, 456 mm, 530 mm). Thus, the number of experimental data was reduced to 44. In Fig. 3a and Fig.3b, a huge variation of the frost density along the plate are observed. However, the mean density measurement is shown as a most representative value of the frost density across the plate.

The identification of the dimensionless groups that affected the frost density was carefully performed to improve the new coorelation. In Fig. 4, the experimental dimensionless frost density ($\rho_f/\rho_i$), which is the ratio between frost and ice density ($\rho_i$= 920 kg/m³), is presented as a function of modified Jakob number (Ja) and dimensionless temperature (T*), which is defined by Eq. (9.1). In Fig. 4.a, the frost density is presented as a function of Jakob number. The dimensionless frost density decrease with the increase of the Jakob number, however, a clear trend is only observed with the experimental results from Hermes et al. (2009). In Fig. 4.b, the dimensionless frost density is presented as a function of the dimensionless temperature (T*), which showed a similar trend like in Fig. 4a. Nevertheless, the data from Lee et al. (2003) seems to follow a well-defined trend. Yang and Lee (2004) also used the dimensionless group, which indicated to have huge potential to correlate with frost density. Additionally, other several groups were tested such as mass transfer Fourier number ($Fo_m$), Lewis number (Le), Schmidt number (Sc), Sherwood number ($Sh_L$), mass transfer Stanton number, but the best prediction was given by the dimensionless groups: Reynolds number (Re), Fourier number (Fo) and modified Jakob number (Ja), dimensionless temperture (T*). The plate length was used as characteristic length. Thus, the new correlation is defined by Eq. (9), which is adjusted from the experimental database presented in Tab. 1.

$$\rho_f^* = 1.54810^{-3}\left(T^*\right)^{-0.096} \mathrm{Re}^{0.3049}\, Ja^{-1.015} Fo^{0.2352} \omega_a^{-0.2943} \tag{9}$$

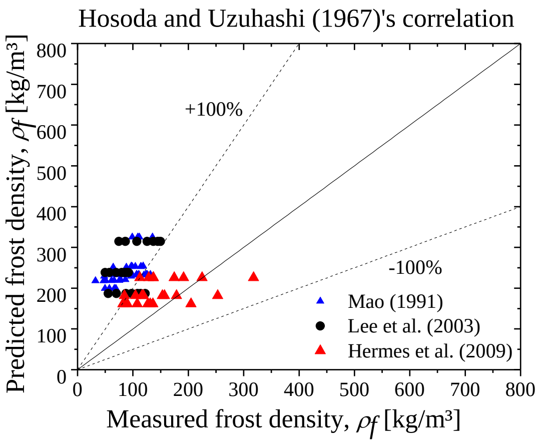
$$T^* = \left(\frac{T_a - T_{tp}}{T_a - T_w}\right) \tag{9.1}$$

## 3. DICUSSIONS AND RESULTS

The performance of frost density models presented in section 1 is evaluated in Fig.5 up to Fig.9. In Tab.1, it is presented the experimental database applied in the assessment of the models. Additionally, the Average Relative Error (ARE), which is defined By Eq. (10), is also applied to evaluate the model's performance. The results are presented in Table 2.

$$ARE = \frac{1}{N}\sum_i^N \sqrt{\left(\frac{\rho_{model} - \rho_{exp}}{\rho_{exp}}\right)^2} \tag{10}$$

a)




b)

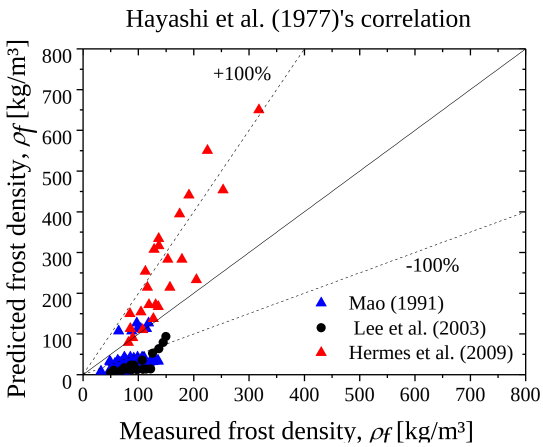



Figure 5. Experimental frost density compared to predictions provided by (a) correlation of Hosoda and Uzuhashi (1967); b) Hayashi (1977).

The correlation of Hosoda and Uzuhassi (1967) is evaluated in Fig.5a. A spreading over than 100% was obtaneid for experimental database presented in Tab.1, which suggested that the wall temperature ($T_w$) and air velocity ($v_a$) are not enough to model the prediction of frost density, as suggested by Eq. (1). Better predictions are provided by the correlation of Hayashi et al. (1977), which showed the Average relative error of 65.4% (Tab.1). The frost surface temperature was a parameter which upgrades the results, however, Hayashi et al. (1977) didn't present a method to calculate it.

The correlation of Mao (1991) was also compared with the experimental results presented in Tab. 1. However, the correlation of Mao (1991) provides local frost density. So, The Eq. (11) is applied to given the average frost density across the flat plate. A greater performance of correlation of Mao (1991) was got leading an ARE of 26.9%. Yang and Lee (2004) also developed a model which presented a reasonable prediction (ARE of 23.2 %).

$$\rho_f^* = \frac{\int_0^L \rho_f^*(x)dx}{L} \tag{11}$$

a) b)

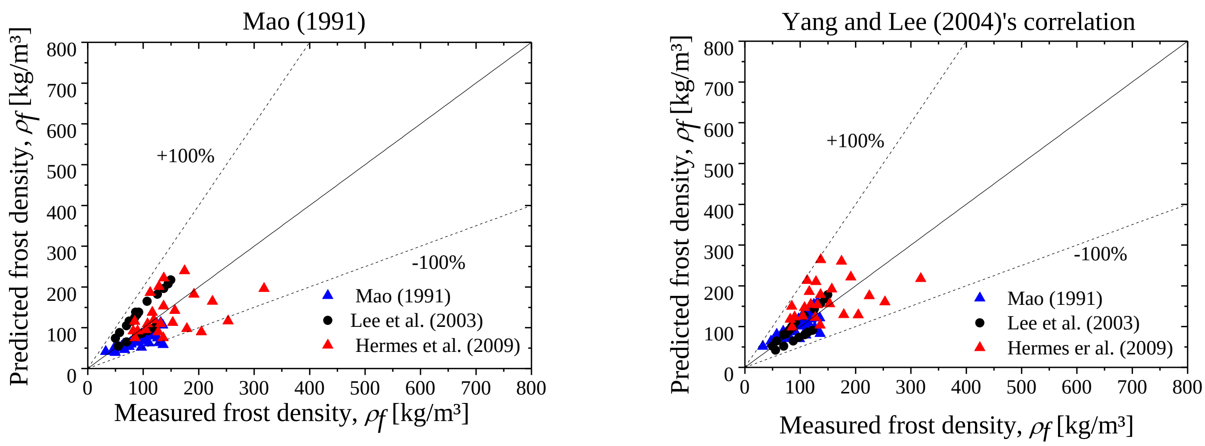



Figure 6. Experimental frost density compared to predictions provided by (a) correlation of Mao (1991); b) Yang and Lee (2004) correlation.

a) b)

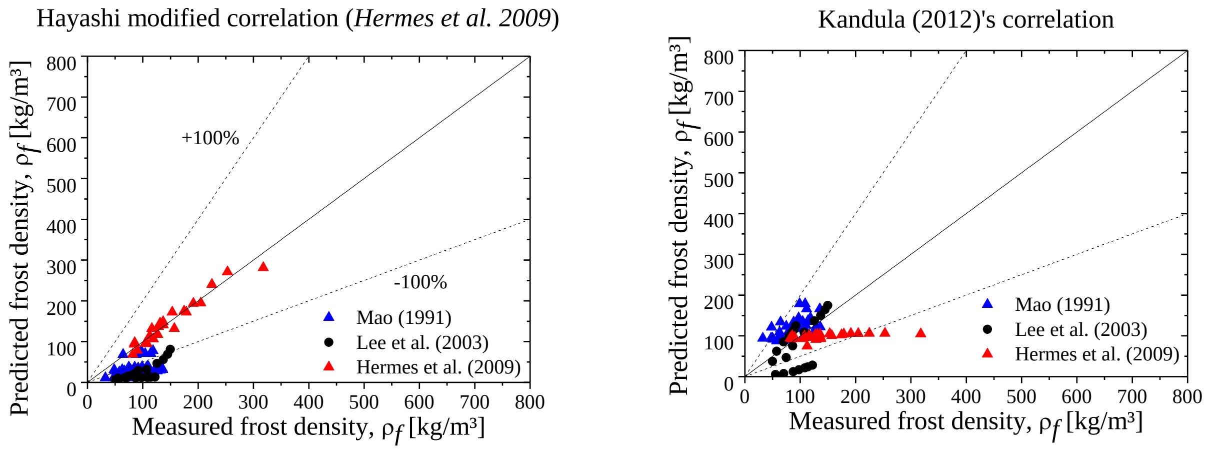



Figure 7. Experimental frost density compared to predictions provided by (a) correlation of Hayashi modified (Hermes et al. 2009) b) Kandula (2012).

Hermes et al. (2009) improved the correlation of Hayashi et al. (1977). The addition of the wall temperature in correlation enhanced the prediction as observed in Fig. 7.a. Moreover, the average relative error (ARE) was reduced to 50.4% (Tab. 1). Fig. 7b. presented the Kandula (2012) correlation, which presented ARE of 38.8%. The semi-empirical correlation developed by Hermes et al. (2014) presented the best performance, with ARE of 32.1% (Fig. 8a). This correlation shows a performance very close with a correlation of Yand and Lee (2004) (23.2 %). Finally, the prediction of Leoni et al. (2016) is presented in Fig. 8.b. unsatisfactory performance was clearly observed (ARE of 114.5%).

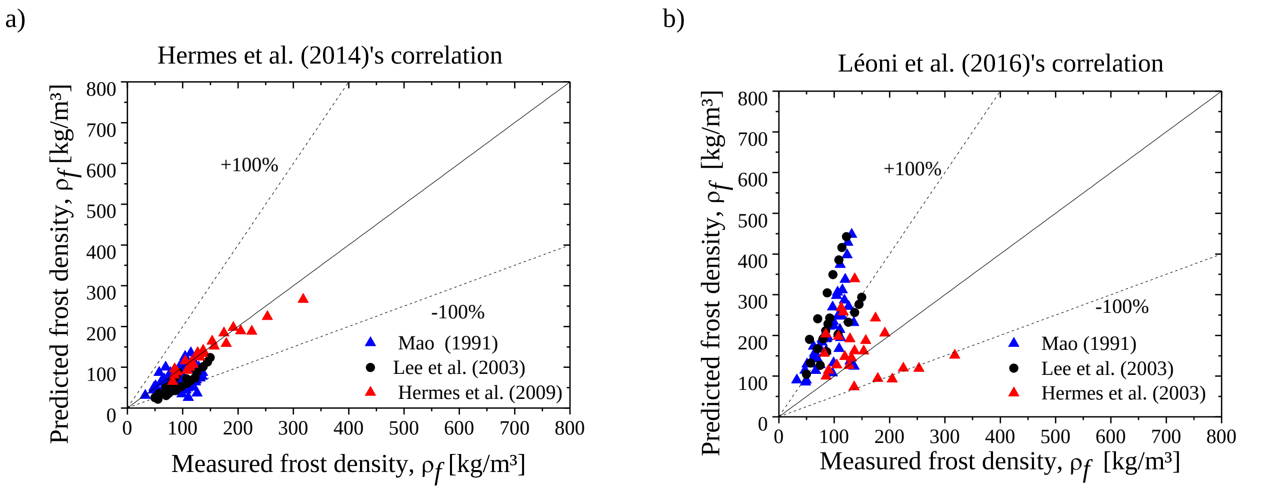



Figure 8. Experimental frost density compared to predictions provided by (a) correlation Hermes et al. (2014) b) Léoni et al. (2016).

Table 2. Average relative errors (ARE) obtained for frost density models found in the literature.

| Model | ARE, % |
|---|---|
| Hosoda and (1967) | 145.6 |
| Hayashi et al. (1977) | 65.4 |
| Mao et al. (1992) | 26.9 |
| Hayashi modified (Hermes et al. 2009) | 50.4 |
| Yang and Lee (2004) | 23.2 |
| Kandula et al. (2012) | 38.8 |
| Hermes et al. (2014) | 23.1 |
| Léoni et al. (2016) | 114.5 |
| Proposed correlation | 18.9 |

The prediction of the proposed correlation is presented in Fig. 9. The experimental data from Lee et al. (2003) and Mao (1991) was properly predicted by the new correlation. However, the database from Hermes et al. (2009) seems not to be exactly predicted by the new model. However, a better average relative error (ARE) is obtained (ARE 18.9) as observed in Tab.2.

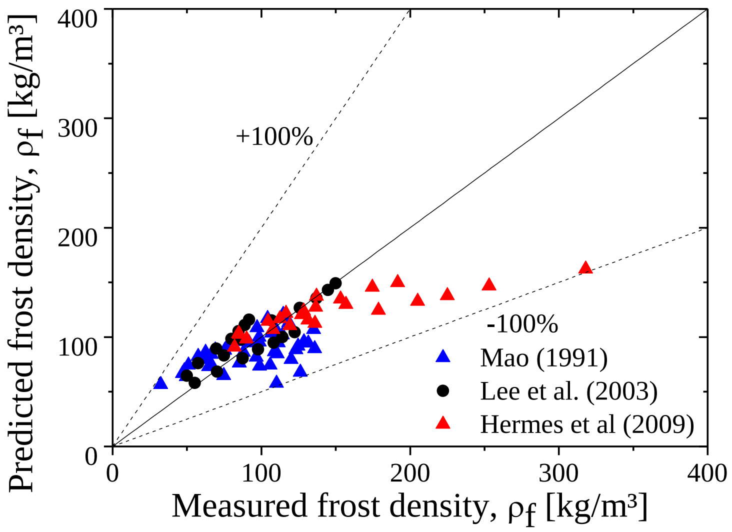



Figure 9. Experimental frost density compared to predictions provided by the proposed correlation.

## 4. CONCLUSIONS

The proposed correlation presented the best performance since the average relative error (ARE of 18.9%) presented the lowest value (Table 2). Additionally, the new correlation has a larger application range (20.7°C < $T_w$ < -5°C and 13.4°C < $T_a$ < 23.4°C) compared against the other models presented in the literature. However, the correlation proposed by Hermes et al. (2014) and Yang and Lee (2004) presented a similar performance, with an average relative error of around 23 %. The Hermes et al. (2004) is a simpler correlation to be implemented, which makes it more suitable for its application range.

## 5. ACKNOWLEDGEMENTS

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## 5. NOMENCLATURE

cp specific heat capacity of air [J kg−1 K−1]

Fo Fourier number

isv Latent heat of desublimation (J kg1) (=2830 kJ kg1)

Ja Jakob number

L plate length [m]

Re Reynolds number

Rec critical Reynolds number,$10^5$

T Temperature [°C]

T Time [s]

v Velocity [m/s]

x distance[m]

Greek Symbols

$\rho$ density [kg/m³]

$\omega$ humidity ratio [ kgv · kga−1 ]

Subscripts

a humid air
i ice f frost
c critical
m melting
s frost surface

## 7. RESPONSIBILITY NOTICE

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